LESSON

6.2 Matrix Operations

OBJECTIVES

- Learn these matrix operations: addition, scalar multiplication, and multiplication
- Analyze matrix dimensions to determine whether they can be added or multiplied
- Use matrices as a tool for describing transformations of geometric shapes

You've seen that a matrix is a compact way of organizing data, similar to a table. But unlike tables, matrices can be added and multiplied to help you solve problems.

Consider this problem from Lesson 6.1. Matrix $[A]$ represents sports drinks, fruit juices, and waters sold this week from the vending machines at the main entrance and the back entrance of the school. Matrix $[B]$ contains the same information for last week. What are the total sales, by category and location, for both weeks?

$$[A] = \begin{bmatrix} 83 & 33 \\ 65 & 20 \\ 98 & 50 \end{bmatrix} \qquad [B] = \begin{bmatrix} 80 & 25 \\ 65 & 15 \\ 105 & 55 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 163 & 58 \\ 130 & 35 \\ 203 & 105 \end{bmatrix}$$

EXAMPLE A

This matrix represents a triangle.

$$\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

a. Graph the triangle and its image after a translation left 3 units. Write a matrix equation to represent the transformation.

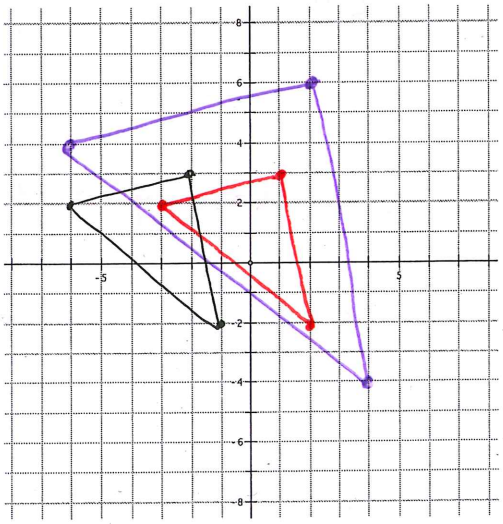
b. Describe the transformation represented by this matrix expression.

$$\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} + \begin{bmatrix} -4 & -4 & -4 \\ -3 & -3 & -3 \end{bmatrix}$$

Translation left 4 and down 3 units.

c. Describe the transformation represented by this matrix expression.

$$2 \cdot \begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$



a.)

$$\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -6 & -2 & -1 \\ 2 & 3 & -2 \end{bmatrix}$$

c.) Horizontal and vertical dilation SF 2

$$2 \cdot \begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -6 & 2 & 4 \\ 4 & 6 & -4 \end{bmatrix}$$

EXAMPLE C

Consider this product.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

2×2 × 2×3 = 2×3

a. Determine the dimensions of this product.

b. Describe how to calculate entries in the product.

$$\begin{bmatrix} R1\times C1 & R1\times C2 & R1\times C3 \\ -1\cdot-3+0\cdot2 & -1\cdot1+0\cdot3 & -1\cdot2+0\cdot-2 \\ 0\cdot-3+1\cdot2 & 0\cdot1+1\cdot3 & 0\cdot2+1\cdot-2 \\ R2\times C1 & R2\times C2 & R3\times C3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 3 & -2 \end{bmatrix}$$

Each row is multiplied by each column.

EXAMPLE B

The school cafeteria offers a choice of ice cream or frozen yogurt for dessert once a week. During the first week of school, 220 students choose ice cream and 20 choose frozen yogurt. During each of the following weeks, 10% of the frozen-yogurt eaters switch to ice cream and 5% of the ice-cream eaters switch to frozen yogurt. How many students will choose each dessert in the second week? In the third week?

NEXT WEEK

	I	FY
THIS WEEK I	.95	.05
THIS WEEK FY	.10	.90

$$\begin{bmatrix} I & FY \\ 220 & 20 \end{bmatrix}\cdot\begin{bmatrix} .95 & .05 \\ .10 & .90 \end{bmatrix} = \begin{bmatrix} 220\times.95+20\times.1 & 220\times.05+20\times.9 \end{bmatrix}$$

$$\begin{bmatrix} 211 & 29 \end{bmatrix}$$

(1×2) × (2×2) = (1×2)

Group Challenge

1. Find the missing values.

x = 12 y = 0 z = 5

a. $[3 \quad 12 \quad -8] + [9 \quad -12 \quad 13] = [x \quad y \quad z]$

b. $-5\begin{bmatrix} 3.8 & -5.2 \\ -1.9 & 0.8 \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$

$$\begin{bmatrix} -19 & 26 \\ 9.5 & -4 \end{bmatrix}$$

2. Perform matrix arithmetic in 2a–d. If a particular operation is impossible, explain why.

a. $\begin{bmatrix} 8.5 & 4.2 \\ 3.6 & -2.7 \end{bmatrix} - \begin{bmatrix} 7.9 & 8.8 \\ 2.9 & -0.9 \end{bmatrix}$ $\begin{bmatrix} .6 & -4.6 \\ .7 & -1.8 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 4 & 5 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ -8 \\ 4 \end{bmatrix}$ (3×3 · 3×1 = 3×1) $\begin{bmatrix} 10+0+8 \\ -10-32+20 \\ 0-8+12 \end{bmatrix} = \begin{bmatrix} 18 \\ -22 \\ 4 \end{bmatrix}$

c. $\begin{bmatrix} 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}$ (1×2 · 3×2)

NOT POSSIBLE – INNER DIMENSIONS NOT EQUAL

d. $0.5\begin{bmatrix} 20 & -10 \\ 16 & 14 \end{bmatrix} + 2.5\begin{bmatrix} 12 & -8 \\ -16 & 30 \end{bmatrix}$

$\begin{bmatrix} 10 & -5 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} 30 & -20 \\ -40 & 75 \end{bmatrix}$

$\begin{bmatrix} 40 & -25 \\ -32 & 82 \end{bmatrix}$

3. Find the missing values.

$\begin{bmatrix} 4 & a \\ b & -2 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 30 \\ 18 \end{bmatrix}$

$4 \cdot 3 + a6 = 30$

$6 \cdot 3 + -2 \cdot 6 = 18$

$36 - 12 = 18$

$36 = 30$

$b = 10$

$12 + 6a = 30$

$6a = 18$

$a = 3$

Write the dimension of each matrix in the space provided .

Let $A = \begin{bmatrix} 1 & 5 \\ 2 & 0.1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 1 & 5 \\ -3 & 1 \end{bmatrix}$ $C = \begin{bmatrix} \frac{2}{5} & 0 & 2 \\ 0 & \frac{9}{2} & 2 \\ 0 & 0 & -\frac{5}{2} \end{bmatrix}$

Dimension of matrix A 2 × 2 *Matrix B* 3 × 2 *Matrix C* 3 × 3

Circle all products that can be found.

AB AC (BA) BC CA (CB)